What's a real 2D shape? Designing appropriate geometric instruction


Mairead Hourigan
Mary Immaculate College,
University of Limerick
<mairead.hourigan@mic.ul.ie>

Aisling Leavy
Mary Immaculate College,
University of Limerick
<aisling.leavy@mic.ul.ie>

Mairead Hourigan and Aisling Leavy describe a range of teaching and learning activities focusing on the identification and classification of 2-dimensional shapes. The activities described are useful in highlighting students' misconceptions regarding non-traditioanl and non-prototypical shapes.

This paper describes the process of designing and teaching an instructional unit on two-dimensional shapes for 5th grade students in two primary schools in the same local school district. Conscious of Van Hiele's (1999, p. 310) promotion of Piaget's tenet that no instruction '…is better than giving it at the wrong time' the first lesson was used to uncover children's level of geometric understanding. This exploration ensured that our geometry instruction was needs-led (Fox, 2000) and built upon current understandings. Therefore activities in the first lesson were carefully selected to ascertain the level within Van Hiele's hierarchy of geometric thinking at which students were located. According to this theory, students functioning at level 0 (the visual level) classify shapes based solely on overall appearance. For example , when presented with a rectangle they may say 'it is a rectangle because it looks like a door'. Students at level 1 (the descriptive level) identify the properties of a and use these for the purpose of classification. For example, when presented with a rectangle, they may classify it as a rectangle based on the number of sides and corners. Students functioning at level 2 (the informal deduction level) can deduce one property from another and make informal arguments to justify their conjecture, for example students may understand that squares are special types of rectangles (Van Hiele, 1999; Robichaux and Rodrigue, 2010). Research has reported that students may 'straddle' levels and that their tendency to judge shapes based on appearance may persist beyond elementary school (Shifter, 1999).

The first lesson consisted of a sequence of open-ended activities. This paper focuses on the first two of these activities, describing the activity as well as the observations made of children's geometric reasoning. The findings with regard to children's knowledge were quite revealing.

Considerations underpinning our selection of 2D shapes

In an effort to determine students' level of geometric thinking, a sequence of activities was developed requiring students to work with polygons (Mack, 2007). As many puzzles, posters, picture books and even elementary level textbooks focus on prototypical shapes such as the regular (all sides and angles equal) triangle and pentagon , we were aware that students may have limited or no experience working with non-prototypical shapes. Research indicates that the overuse of prototypical shapes can lead to difficulties in recognising particular shapes as belonging to a specific category (Fox, 2000; Edwards and Harper, 2010). For example, the first triangle in figure 1, the equilateral triangle, is a common triangle prototype. However if children are exposed only to this prototype, they may not recognise that the other shapes in figure 1 are triangles. Similarly, the over representation of shapes on a horizontal base leads to similar difficulties. The repeated presentation of a square on its horizontal base (see first shape in figure 2) may result in many children not recognising the second shape as being a square.

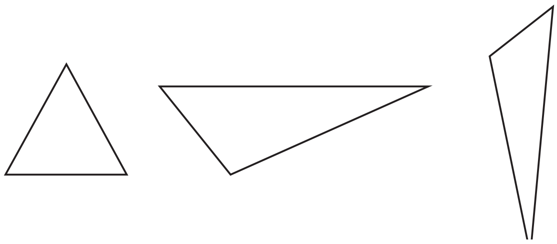

Figure 1: Prototypical and non-prototypical triangles

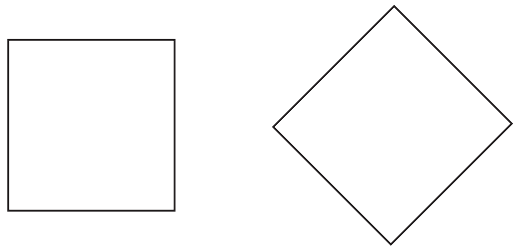

Figure 2: Squares in prototypical and non-prototypical orientations

Therefore we chose to include non-prototypical shapes in our activities. It was intended that the activities would assess and extend students' geometric understandings. An important aspect of these activities is that they encourage children to communicate their reasoning which, in turn, allows us to make more informed instructional decisions when planning subsequent geometry lessons.

Activity 1: Taking a virtual tour

To introduce the lesson, children were invited by the teacher to go on a 'virtual' tour of their school in the search for polygons:

> One of my favourite hobbies is photography. I love taking photos of just about anything. I was asked by the principal to take photos of different areas of the school as she is creating a new brochure for parents. So I went around the building—inside and outside—taking photographs of *everything*! When I was looking back over them, I was surprised to notice that there were a lot of shapes in the photographs; lots of interesting shapes that I had never noticed before. I am going to show you some of these photographs and ask you to identify any different shapes; any unusual shapes that stand out to you. I want you to draw as many shapes as you wish on your response board. Remember! You don't need to know the names of these shapes! Just draw them and when you are finished, describe the shapes.

The teacher displayed a number of carefully selected photographs of everyday objects from the environment on the interactive whiteboard (IWB) (see figures 3 and 4). For each photograph presented, children were encouraged to carefully examine it for interesting shapes. They were invited to draw as many shapes as they wished (see figure 6) and then talk about each shape with their partner. Several children were invited to share their discoveries and descriptions with the class (see figure 7). In their descriptions, children were encouraged to focus on the features and properties of the shapes.

Figure 3: Sample photograph

Figure 4: Sample photograph

Figure 5: Pupils used shape names only

Figure 6: Pupils drawing ‘interesting’ shapes

Figure 7: Teacher discussing children’s drawings

Many children were extremely uneasy with non-prototypical shapes. Apart from a few exceptions (figure 6), children drew only the common prototypical shapes such as rectangle, triangle (see figure 7). During informal conversations with individuals and pairs during this exploration, a range of abilities was evident in descriptions of the properties of 2D selected shapes. Although children were reassured that they did not need to know the shape names, some elected to write the shape names rather than draw the shapes (see figure 5). When describing shapes, while many students referred to properties (number and type of sides, number and type of angles, symmetry and perhaps the name of the shape), others struggled to respond. Many children focused on the overall look of the shape. For example, when describing the isosceles trapezoid (see figure 3 (green shape) and 8a), one child reported “It looks like a triangle with the top cut off”. Another child, when referring to the non-prototypical pentagon (Figure 4 (green shape), 6 and 8b) explained “I cannot describe it because I do not know the name of it”. Some children in their descriptions of this shape described a real world referent that resembled the shape. For example while some reported the non-prototypical pentagon was “like the side of a house” and “like a candle”, others searched for familiar shapes stating “It looks like a triangle and a rectangle put together”. Again children who drew the irregular quadrilateral (see Figure 4 (blue shape) and 8c) made comparisons with more prototypical shapes through statements such as “It’s a slanted square” or “It looks like a square but it’s not a square”.

Figure: 8A

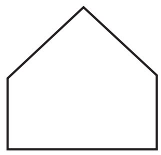

Figure: 8B

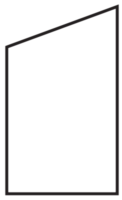

Figure: 8C

When a 5th grade child was asked to describe the triangle she had drawn (see the red response board on Figure 7), her initial response was “It’s like a slice of pizza”. Many children’s responses indicated that they did not consider that it was a ‘real’ triangle: “That’s not a proper triangle” (Mark). When further clarification was sought Mark explained “It’s an upside down triangle”. The teacher used this comment to initiate a whole class discussion by asking the children to compare the 2 shapes (see Figure 7). While all students agreed that the shape on the blue response board was a triangle, there was some support for Mark’s belief that the shape on the red response board was not a triangle. When justifications were requested, students communicated issues with the orientation of the shape:

Charlie: It's not normal because there's supposed to be a point up there at the top and there and there. [Points to two points below the 'top' point.]
Megan: It's supposed to have one point on the top and 2 points on the bottom.

It was necessary for the teacher to ask pupils to consider the properties of a triangle. She recorded their ideas on the IWB. She then systematically went through each of the properties to check whether the shape on the red response board met these criteria. It was only then that all the children recognized that the shape was a "normal triangle". These responses suggest that many of the children struggled with shapes that did not fit the prototype exactly in terms of appearance.

In addition to uncovering children's understandings, the ensuing whole class discussion served to model how a shape can be described by making reference to its various properties. As a result, it was noted that each time the children were presented with a new photograph to search for interesting shapes; students were more focused on the properties of the shapes and less preoccupied with its name or real world referent. This is shown in the following dialogue.

Teacher: Figure 3 is displayed on the Interactive White Board. Ok I can see lots of interesting shapes. Samson, will you show us your response? I am looking at this shape here. [Teacher pointing to one particular shape on the response board.] That's interesting; where did you find this in the photograph? [Samson comes to the IWB and points to the location].
Samson: In the middle here? (See red shape highlighted on Figure 3)
Teacher: Tell me about that shape. Describe it for me.
Samson: It's a diamond.
Teacher: A diamond. Ok, anything else you can say about it?
Samson: It has 4 sides.
Teacher: Anything else you notice?
Samson: Am… [silence]
Teacher: Can anyone help him? Michael?
Michael: It's also a rhombus
Teacher: A rhombus!! So what do you know about a rhombus?
Michael: It's sort of like a square squashed out of shape
Teacher: Squashed out of shape…and why do you say that?
Michael: It's kind of slanted
Teacher: And can you say anything else about these sides? Look very carefully at this shape. What else could you say? [silence] Maybe look at this side and this side [pointing towards opposite sides]
Michael: They are parallel
Teacher: They are parallel. Great. What does parallel mean Martha?
Martha: The lines will never meet.
Teacher: And how many sets of parallel sides does a rhombus have? Tomiek?
Tomiek: 2?
Teacher: 2—exactly—these 2 here and these 2 here [pointing to the photograph]

This activity provided much valuable feedback to the team in planning the subsequent lessons. The observations suggested that many children had little experience of non-prototypical shapes. In lessons which followed it was important to include a range of shapes which would encourage children to consider shapes outside the regular, familiar shapes they seemed to have been limited to. It was decided to prioritise the focus on shape's properties as opposed to names only.

Activity 2: Classifying shapes according to rules

For this activity, 26 shapes (both prototypical and non-prototypical) were selected for inclusion in the set of shapes (see Figures 9 and 10). It is necessary to have large shapes (teacher and pupil demonstration (see Figure 9) as well as smaller shapes for each pair (see Figure 10). Shapes were numbered to help with identification and selection during the activities (see Figure 11).

A game called What's the rule? introduced the idea of classifying shapes according to a certain property. Each child was given a large (numbered) shape. The teacher called out a series of particular numbers and asked for students with those shapes to stand at the front of the classroom. The remaining children had to work out what property the selected shapes have in common. For example, the teacher stated "If these 3 shapes are part of the same group, 'what's the rule' for the group?" (see Figure 9). This activity was completed a number of times.

Figure 9: Students holding shapes selected for 'What's the rule?'

This was followed by the introduction of property loops (cf. Oberdorf, 1999), which were referred to as 'rule rings'. The teacher demonstrated how a 'rule ring' can be used to classify shapes according to a particular rule. Children selected a rule for example "all shapes with 3 straight sides", and in consultation with the class the teacher implemented this rule using the rule ring. Children became aware that all shapes that met the rule were placed inside the ring whereas those that did not were placed outside the ring. A set of shapes and rule rings were distributed to each pair and the teacher instructed them to create and implement their own rules (see Figure 10). This open-ended activity challenged children to explore shapes and identify properties that they have in common.

While some pairs initially depended heavily on rules relating to the number of sides, once encouraged to select another type of rule, they quickly demonstrated a variety of sorting criteria. Another popular sorting criterion was angles (e.g., shape with a right angle).

We were equally accepting of both formal and informal language such as "Our rule is shapes with pointy corners". Children were encouraged to explain their rule "What's a corner?" and the teacher sought to build on the responses to introduce and reinforce the formal mathematical term "Oh your rule is shapes with an acute angle? Great". On regular occasions during this activity, pairs were invited to demonstrate the placement of their shapes (using the shapes on the IWB) so that the class could guess the rule. Alternatively pairs were selected to share their rule with time allowed for the class to implement this rule.

Figure 10: Pupils classifying set of shapes using rule rings

After the 'free-sort' activity using the entire set of shapes, the teacher asked the students to gather together a collection of particular shapes (see Figure 11). Shapes were pre-selected before teaching in order to target particular conceptsand predict the rules the children may select for example regular vs irregular shapes, symmetry vs no symmetry. Children had to identify the property that some of the shapes had in common and place all others outside the ring. It was interesting to note that after exhausting the obvious rules such as number of sides, types of angles, symmetry, some of the classification rules focused on appearance for example "looks like a square" or "they have a triangle in them".

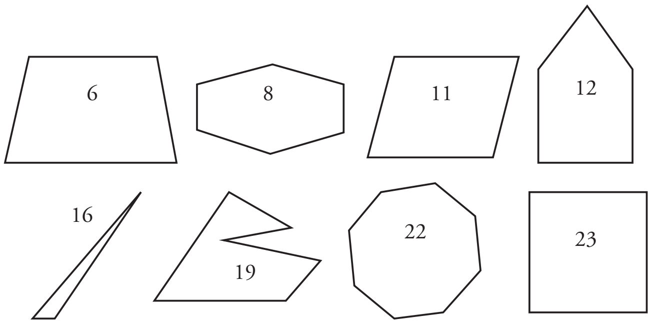

Figure 11: Shapes selected for focused classification

Ben: Could we say these [pointing to shapes 22 and 23] are real shapes?
Teacher: What do you mean by a 'real shape'?
Ben: Ones that people use in maths
Teachers: Ones that we use in maths?
Ben: … And buildings
Teacher: Ok. Well we are in maths class now and we are using these shapes

(pointing to irregular shapes). Why wouldn't you call them real?

Tomiek: Because they don't have a name

Teacher: They don't have a name?

Ben: They look more like stuff that just has been randomly made

Teacher: Randomly made. Can you think of another name, instead of saying they are not real shapes? What else could we call them?

Ben: Inconclusive shapes

Teacher: Inconclusive. Have you ever heard of the term 'irregular shapes'?

Boys: Yes

Teacher: So they are still shapes, they are just not regular like a square

In both classes, many children struggled to recognise that non-protypical shapes had properties which made them part of a shape family. For example, children found difficulty recognising that shape 2 (figure 11) is a quadrilateral and similarly that shape 12 is a pentagon. They also seemed unfamiliar with the terminology 'irregular'. This resulted in many different informal labels being used. For example one pair created a rule "even and odd shapes" and on probing explained that "They are odd shapes because they are uneven… they have different sides and angles" (Tilly). In the other class, in response to the teacher's request for a new rule to those previously used, Ben spoke of 'real shapes'. This discussion below was particularly enlightening on all fronts:

Reflections

The activities described in this article were designed and implemented to gain information which would facilitate appropriate planning and instruction in order for the children to advance in terms of their geometric thinking (Van Hiele, 1999).

Observations suggest that a proportion of children, despite being in Grade 5 were demonstrating characteristics associated with level 0 geometric thinking. The observation that many students were unfamiliar with and unable to deal with non-prototypical shapes suggest that they received little experience non-prototypical shapes or perhaps that little emphasis was placed on non-prototypical shapes in their geometry instruction (Van Hiele, 1999; Robichaux and Rodrigue, 2010; Edwards and Harper, 2010). As a result students often resorted to level 0 behaviours such as 'it looks like…'.

In order to move beyond this basic level of understanding, children need to have opportunities to engage in geometry activities that incorporate a wide range of non-traditional and non-prototypical shapes. Children need to be challenged to move beyond shape recognition and focus on the characteristics of these shapes i.e., their properties. This focus should be ongoing from Kindergarten. As appropriate instruction is deemed essential to development, the selection of activities will prove crucial. Activities may be as simple as shape searches (in the classroom, school, home etc.), comparing shapes (What is the same?, What is different?), construction activities (making pictures with shapes, making shapes) and shape sorts; activities wherein children are encouraged to manipulate, record, describe and justify (Van Hiele, 1999). Then children will be ready to focus on classes of shapes such as 'quadrilateral' and in time use this knowledge to engage in informal deductive reasoning of conjectures such as 'All quadrilaterals are parallelograms—True or False?'

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