Exploring probability concepts using a mini games environment: Ideas for primary and secondary classrooms

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Introduction

An understanding of the language of probability is important to function in society and become critical consumers of the data presented in newspapers, surveys and consumer reports. Such understandings help students make decisions in uncertain situations ranging from everyday events (such as deciding whether to wear a raincoat and ascertaining the likelihood of rain) to the outcome of sporting events (examining league tables to determine the likelihood of a sports team winning a game). On a daily journey students are faced with uncertainty in a variety of guises: what is the likelihood of making most of the twelve lights on the drive into school today? Will we get across the road before the green light turns red?

However, understanding probability can be a challenge. Children's everyday experiences and intuitions about probability often pose obstacles to developing a correct understanding of probabilistic concepts (Fischbein 1975; Shaughnessy 1992). Godino and Batanero (2004) state that probability can be "a notoriously difficult topic to teach even for high school and college students in part because students' intuitions are not always in sync with the mathematics of probability".

Let's take the example of *fairness.* In mathematics, the fairness of a game is associated with players having an equal chance of winning. However, an everyday meaning of *fair* can deviate from the mathematical meaning. For many students when playing games, *fairness* does not refer to whether the game is mathematically fair but rather to the behaviour of players while playing the game. Hence a student may believe that if all players play according to the rules, and don't cheat, that everyone has an equally likely chance of winning. Students may not associate fairness of a game with the chance of winning – this has implications for developing robust understandings of fairness and in turn probability. It is important for teachers to be alert to possible confusion for students when everyday and mathematical meanings are different.

Learning Objectives

The activities that we describe are designed to provide a range of games-based experiences, which, in conjunction with carefully orchestrated teacher discussion, help students understand *what it means for a game to be fair*.

Depending on the class being taught, we provide suggestions on how to extend the learning goals of the activities and help students relate probability to experimental outcomes and explore how probability is related to fairness.

Designing and implementing the series of lessons

These activities were designed and researched in upper primary classrooms as a part of the Lesson Study research in mathematics education carried out at Mary Immaculate College (Leavy 2010; Leavy et al 2013). The games can be explored as stations activities in your classroom using an arcade type scenario described below. Another option, but more resource intensive, is to have all students play each game at the same time, which may allow for more focused discussion on the fairness, expected probabilities and outcomes of each game.

Launching the lessons

In an effort to situate the explorations of probability within a real world context, we showed a video clip of 'Caine's Arcade' [http://cainesarcade.com/]. Cain is a 9 year old boy who built an elaborate cardboard *arcade* inside his dad's auto parts store (Figure 1). In the video excerpt Cain explains that some of his arcade games are easier to win than others. Cain uses his own toy cars as prizes. However, as Cain explains, he has only got a certain number of cars, and if every customer wins every game every time, he would run out of prizes very quickly. After watching the video, encourage students to discuss how Caine designed the games so that they would not be too easy.


Figure 1: Cain's Arcade

Tell students that they are going to play a number of games in small groups. Then present the class with this challenge:

> *Caine is always adding new games to his arcade. We know that he needs games that are possible to win. However, the chances of winning cannot be so high that he runs out of prizes. By the end of today, you are going to recommend a game that would be suitable for his arcade. To do this well, we need to experiment and explore lots of different games to find out more about chances of winning so we can help Caine.*

Here are some general guidelines. We provided recording sheets for each game (see Figure 4 for sample).

***Before* playing each game.** Encourage students to predict the outcome of the game. What colour/number/card is likely to win? Why? Older students are encouraged to determine the probability of winning the game and record it (i.e. the theoretical probability).

While playing the game. Encourage students to record the events (i.e. the actual outcome).

***After* playing the game**. Encourage students to think about the *fairness* of the game. Is it a fair game? Why? Why not? How might you make it more (or less) fair? Then, discuss the suitability of the game for Cain's arcade i.e. how many toys Cain might have to give away if this game featured in his arcade.

Note: For younger students do not expect them to initially make mathematically sound predictions. Older students can be encouraged to make a list of possible outcomes and use more probabilistic language in their predictions, discussion and explanations.

The Games

These games use common materials and can be easily modified to suit the abilities of your students and your required learning outcomes.

Game 1: *The 12 sided die*

In this game the group roll a 12 sided die (Figure 2). They must roll a 1 to win. Students first calculate the probability of winning (1 in 12) and then roll the die 12 times and record how many times they rolled a 1.


Figure 2: A 12 sided die

Game 2: *Find the King*

The group is presented with 3 cards. One of the cards is a King – you must find the king to win (Figure 3). A dealer shuffles the cards and places them face down on the table.

Students calculate the probability of winning (1 in 3), play this game 3 times and record how many times that they find the King.

Figure 3: Find the king to win

Game 3: *Bag of Cards*.

This station has a bag with 9 cards. There are 2 Aces and 7 other cards. Students have to pick a card from the 9 cards hidden in the bag. They must pick an Ace to win (Figure 4). They calculate the probability of winning (2 in 9) and then play the game 9 times, replacing the selected card each time. Students record how many times they find an Ace.

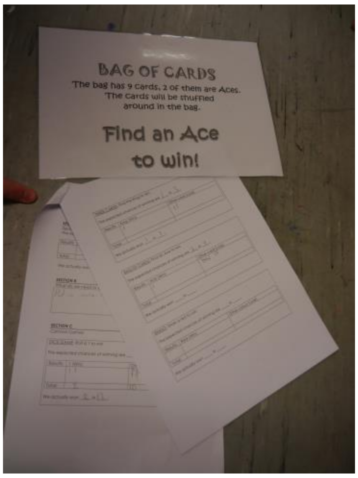

Figure 4: Find an ace to win

Game 4: *Let's play Bingo*

In this activity a bag contains 12 counters: 10 red and 2 green. Students have to draw a red counter to win. They calculate and record the chance of winning (10 in 12). They take a counter from the bag, record the colour, and then replace it. They make 12 draws. Student record how many times they draw a red counter.

Game 5: *Lottery*

A bag is filled with 9 numbered ping-pong balls. Students must draw a 9 to win (Figure 5). They record the chances of winning (1 in 9) before making 9 draws. They then record the number of times they drew a ping pong ball with the number 9.

Figure 5: Lottery: Get a 9 to win

After playing each of the games, return to discuss the focus question - what is the 'best' game for Cain's arcade? It will soon become apparent that different groups had different outcomes from the same games. This presents an opportunity to investigate the hypothesis that the more times you carry out the activity, the closer your actual results are to the expected/predicted results. Tell students that if we play each game more times, we might get a better idea of whether or not it is a good game for Cain's Arcade – as his games will be played more than 12 times.

Finding the best game for Cain's arcade: What happens when we increase the number of trials?

Invite each group to revisit one game and play that game 36 times. Have them examine their results the first time and then predict their results if they played the game 36 times. Once finished, compare both results. Primary level students may need support comparing 'equivalent fractions' and a fraction wall would support the necessary comparisons. Use the following questions to guide discussions:

- What differences did you notice between the first time you played the game and when you played it 36 times?
- Which results were closer to the chances we expected: the 1st time? 2nd time? Why do you think that is? Discuss this in your groups and come up with a reason why.

Many groups will find that when they played the game 36 times, the chance of winning the game was closer to what they expected as compared to when they played the game the first time. Younger students may refer to the role of 'luck'. Older students may come to the conclusion that the greater number of tries brings the actual (experimental) outcome closer to the theoretical probability.

To conclude the lesson, have each group write or present a short justification **for the game that they think should be included in Cain's arcade**. Justifications will differ depending on class level and may range from references to the outcomes from their games to reporting the theoretical probabilities associated with winning each game.

Expected student learning

Experiences with random-producing materials (dice, spinners, cards) supports students in developing their concept of probability by discussing the likelihood of different events. Students should come to realise that even though they can calculate the probability they can never know exactly what will happen next in the game, but they get an idea about what to expect. As they play games, results are recorded and those results used to make predictions about the best game. Students should learn in which game there is a better chance of winning and whether games favour the player or the arcade owner. They should be able to provide the data (i.e. probabilities) as a justification for their decisions. The activities should lead to a discussion of fairness of games, the potential difference between the expected and actual outcomes, and the impact of increased trials on this difference.

Extensions for older students

The discussions arising from these games lay the foundations helping students determine and compare experimental and theoretical probabilities for simple and compound events. Encourage students to design their own games with predetermined outcomes i.e. making their own spinners. Design investigations to determine the fairness of everyday games such as *rock*, *paper*, *scissors* and use tree diagrams for determining the probability of each outcome (Nelson & Williams 2009). In this online game, a virtual die is used to direct the distance a car travels and in turn to motivate the notion of conditional probability and the notion of random numbers (e.g. http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/).

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