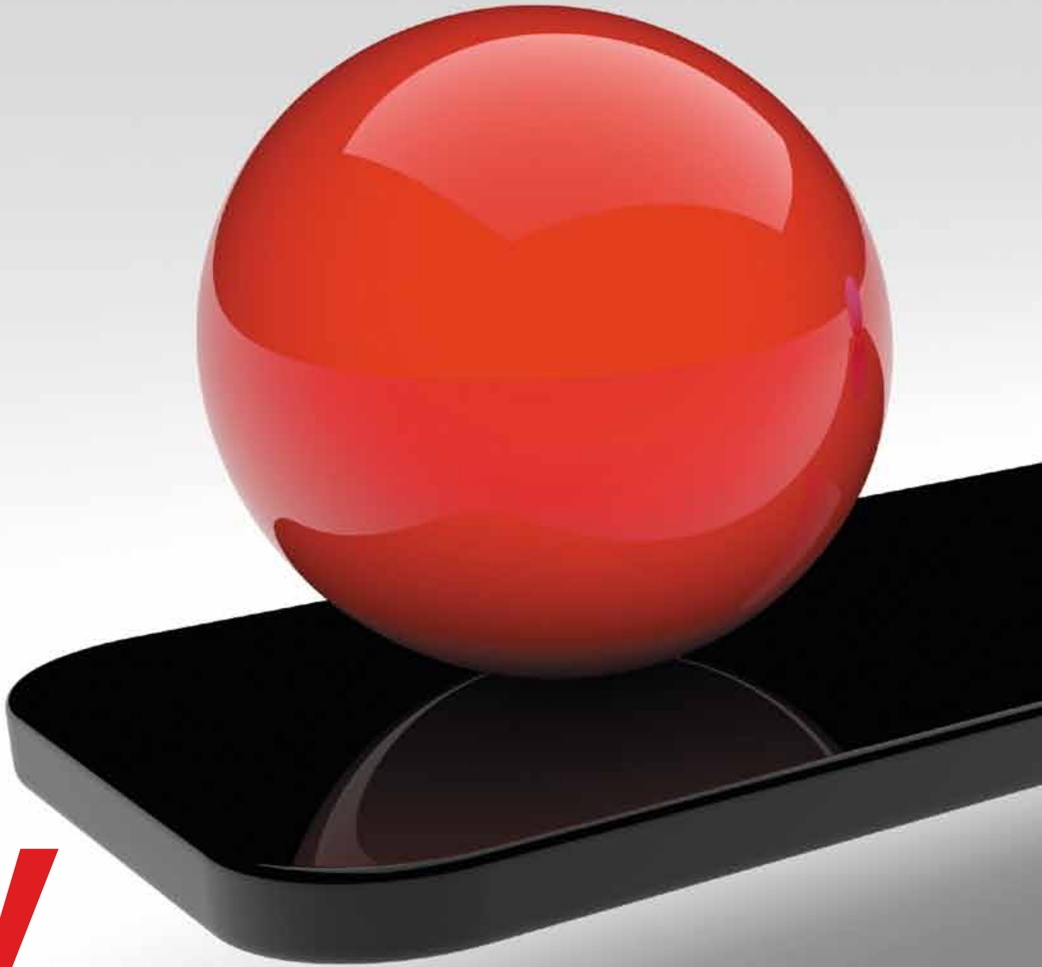
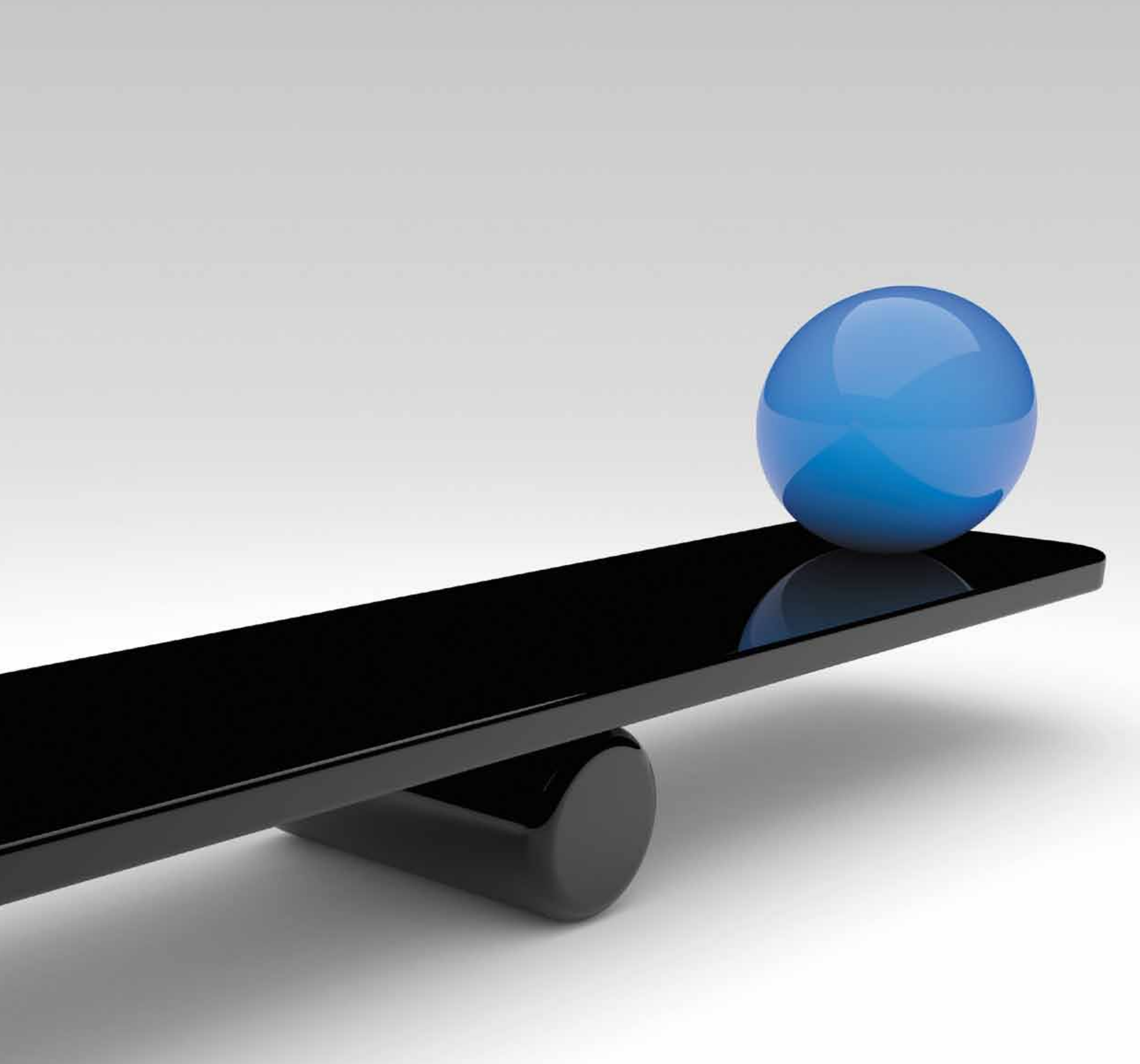


Early Understanding of Equality

One of the first math symbols introduced =the equals sign= underpins much of the algebraic reasoning a child will use in later years.



LCS/VEER

By Aisling Leavy, Mairéad Hourigan, and Áine McMahon

Quite a bit of the arithmetic in elementary school contains elements of algebraic reasoning. After researching and testing a number of instructional strategies with Irish third graders, these authors found effective methods for cultivating a relational concept of equality in third-grade students.

Why so important?

Understanding equality is fundamental to algebraic understanding. Although we introduce the equals sign to children as young as five or six, equality is not simple. The notion of *equal* is "complex and difficult for students to comprehend" (RAND Mathematics Study Panel 2003, p. 53). Overcoming such difficulty is essential because an understanding of equality supports the transition from arithmetic to algebra and forms the basis for comprehension of equations and inequalities. The huge impact that understanding equality has on later algebraic understanding draws attention to the importance of identifying and tackling misconceptions that are related to equality when they arise in elementary school classrooms.

Operational or relational?

To investigate understandings relating to equality in your classroom, present the following problem:

8 + 4 = □ + 5.

The two responses below indicate an operational view of equality:

8 + 4 = **12** + 5
8 + 4 = **17** + 5

This operational view may be rooted in elementary school experiences that have a proliferation of problems of the form $a + b =$ □. Seeing multiple problems of this form may reinforce the “do something signal” (Kieran 1981, p. 319). This misconception causes difficulties when children encounter later algebraic work that does not conform to this layout and instead requires relational understanding of the equal sign.

Children with a relational view of equality believe that the equals sign means *is the same value as*. They understand that the amounts on either side of the equals sign are relationally the same. They then search for a value that when placed in the frame □ will result in both sides of the equals sign balancing each other. The following (correct) response indicates a relational view of equality:

8 + 4 = **7** + 5

We want children to think relationally about equality. A relational view allows children to think about *relationships between quantities* rather than the quantities themselves.

How do we encourage relational understanding?

We can use a variety of strategies to help children develop a relational understanding of equality. These range from the use of concrete apparatus, such as pan balances and visual images to represent equality relationships (Ellis and Yeh 2008), the representation of missing numbers, the use of technological tools (Jones and Pratt 2006), and developing kinesthetic notions of balance (Mann 2004), to name but a few. We found a sequence of nine approaches that support developing a relational understanding of equality in third-grade classrooms (see **fig. 1**). These approaches, presented in the order in which they were taught, were refined by

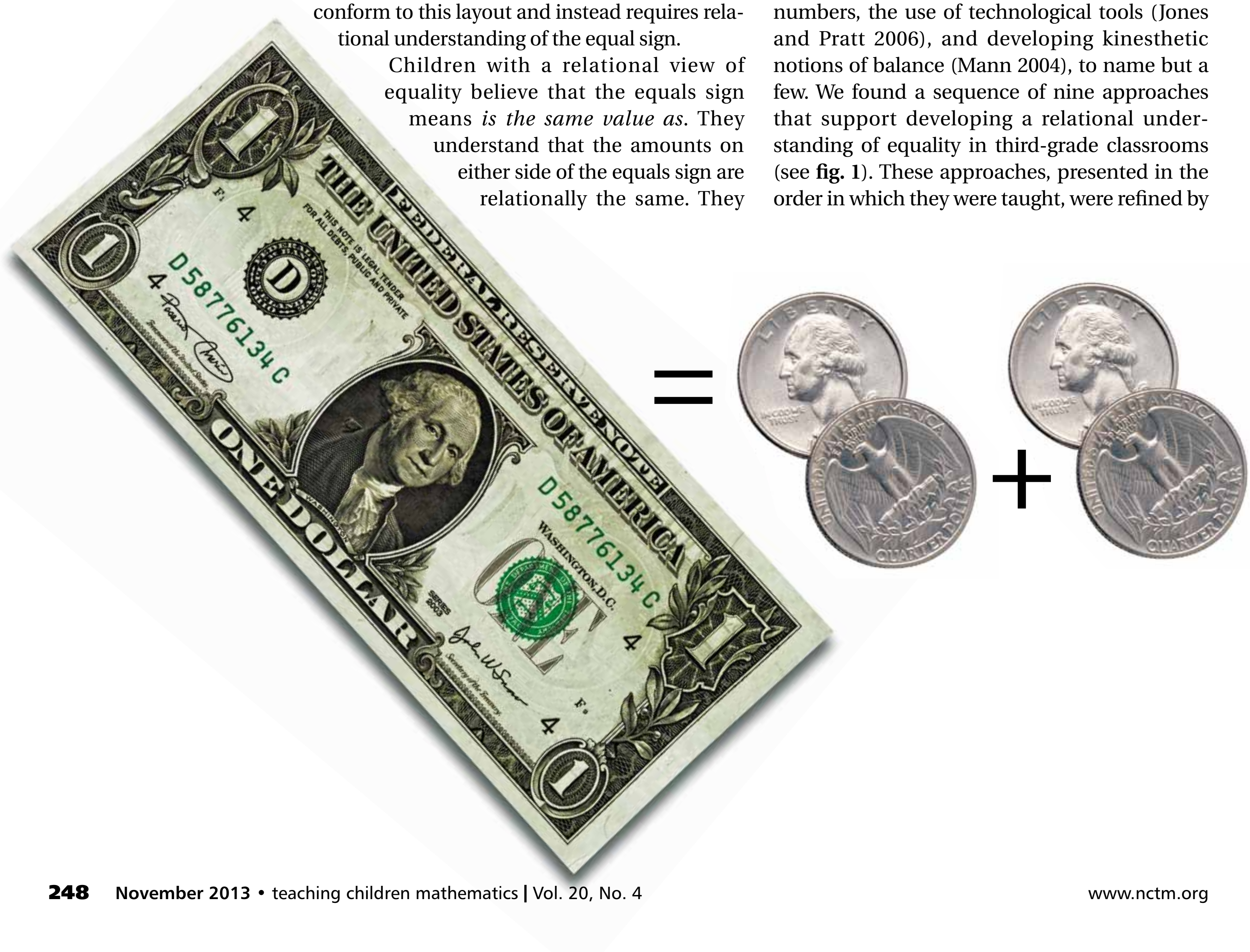
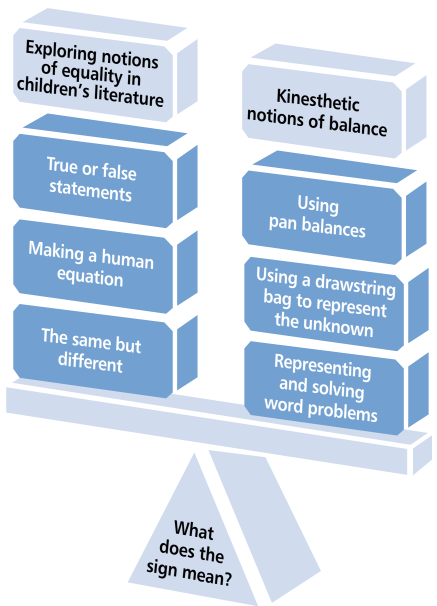


FIGURE 1

The authors worked with school children in Ireland to refine nine approaches to developing notions of equality.



working with children in several different school settings in Ireland.

1. What does the sign mean?

To begin, the teacher presented an image of the equals sign and asked, "Can anybody, if you had to explain the equal sign to me, tell me what it is?" Answers to this initial question can provide valuable insights into whether children have a relational or operational view of equality. Once asked, the children needed some time to think about the question and express their thoughts in a coherent way. Even as adults, we can find it hard to explain what we believe the equals sign to *mean*. In general, more than half the third graders held an operational understanding. One responded, "It's the thing going to the answer that the two numbers equal."

In contrast, a child with a relational view responded, "It actually means 'is the same as,' because one plus one is the same as two."

2. The same but different?

We then emphasized that expressions may *have the same value but look different*, which reinforces the idea that equality means balance; balancing involves the same value on both sides of the equals sign, even though both sides may look quite different. Many contexts can be used to illustrate this idea. We selected the context of money for a number of reasons. First, third-grade children could relate easily to the context. It is also a helpful context in reinforcing the idea that a variety of different combinations may still have equal value. Definitive examples include the following:

$\$1 = 50¢ + 50¢ \quad 25¢ \times 4 = \$1 \quad 50¢ + 50¢ = 25¢ \times 4$

Children did not have any difficulty with this approach and readily engaged in the discussion. We found that the use of money was effective, providing an almost tangible crutch for children to grapple with the notion that quantities that appear different may have the same value. The following excerpt was developed from children's responses during the first lesson.

Teacher: So, if "equals" means "is the same as," does it mean the same for a sum like $3 + 5 = 4 + 4$?
Anna: It means "the same as" because $3 + 5$ is 8, and $4 + 4$ is 8, and 8 is the same as 8.
Teacher: Supposing I write [*writing on the board*] 1 Euro = 5×20 cents. Does the equal sign mean "the same as" then?
Ciara: Yes, because 1 Euro is the same as 5×20 cents.
Teacher: OK, but here is a Euro coin, and here are five 20¢ coins. Do they look the same?
Ciara: No, but they add up to the same.

3. Could we make a human equation?

We explored the relational concept of equality further by developing human equations. The teacher had A4-size signs representing the equals sign and the operational symbols. Children were selected to stand at the front of the classroom. They positioned the equals sign and one of the operations signs in a way that made the (human) quantities balance around the equals sign.

For the second problem, the teacher arranged students so that the number of children on both sides of the equals sign was not the same. We placed six students on one side of the equals sign and five on the other side. The class had to investigate which operation must be inserted in

A simple game of tug-of-war can help children develop the idea of balance.

the equation to make both sides balance. (We expected the response $3 + 3 = 3 \times 2$.)

Teacher: I have six equals five. Is that right?
Children: No.
Teacher: How can I make both sides have the same value? What sign must I give Oscar to hold?
Monica: Plus one [to the right-hand side].
Teacher: If we added one. OK, so we could put plus one at the end [*gesturing to the end of the line of children*], and we'd have six equals five plus one. What else could we do?
Keith: Times two by three.
Teacher: Good. So, we'll give Oscar a different sign this time [*handing Oscar a multiplication sign*].
Keith, will you come up to the board for me and write out that sum?
Keith: Yep [*writing* $6 = 3 \times 2$ *on the board*].

4. Exploring notions of equality in children's literature

We then used children's literature to explore equality. We read and discussed the story *Equal Shmequel* (Kroll 2005) about a mouse and her woodland animal friends who decide to play a friendly game of tug-of-war. The central dilemma focuses on how to make both sides equal so that the game is fair. Suggestions range from categorizing the animals into meat-eaters and plant-eaters to ensuring an equal number of animals on each side. None of the solutions work; the bear and some of the bigger animals make the sides unequal because they have more pulling power. The storybook characters solve the problem by distributing animals on either side of a teeter-totter until both sides balance. After they use the teeter-totter to establish equality (of weight), the tug-of-war can begin.

This story served two purposes: (a) we used the story context to introduce the seesaw (or teeter-totter) to explore relational notions of equality and (b) using a variety of different sizes of animal characters (such as a bear and a mouse) to balance the sides reinforced the notion that things can be the same value but at the same time look different. Images from the book were projected on the wall as the teacher read the story aloud.

5. Kinesthetic notions of balance

As the *Equal Shmequel* story of a tug-of-war between woodland animals was being read aloud to the class, children communicated their own predictions regarding who would win by using their hands to demonstrate a kinesthetic notion of balance. The teacher instructed, "Put your hands like a scale or a seesaw [*positioning her hands like a balance*]. Every time I ask if the characters in our story are equal, show me—using your hands—if they are equal or not." This activity reinforced the idea that the value of quantities on either side of the equals sign must balance each other.

6. Using pan balances

To develop balance notions of equality, we had the children use pan balances to solve number sentences by determining the value that would balance both sides. In the excerpt below, a group of third graders placed fifteen cubes on one side of a pan balance and nine cubes on the other side. They explored what they must do to make both sides balance. We expected a solution of $15 = 9 + 6$.

Teacher: So, we have fifteen cubes here and nine here. Which one is heavier?
Children: This one [*gesturing to the side that has fifteen cubes*].
Teacher: OK. So, what do you think you are going

to have to do to make both sides equal? To make them balance? [*She gestures with both hands to convey a balance.*]
Alanna: Add six more here [*pointing to the side with nine cubes*].
Teacher: Why don't you add six more cubes, and we'll see what happens.
Catherine: Or take away six cubes from here [*pointing to the side with fifteen cubes*].
Teacher: You could take away. Yeah, you could do that as well. So, if we take away six cubes from here, what would happen? Let's try.
Catherine: It'd balance [*as students remove cubes from one side*].
Teacher: So, we have nine on each side, don't we? Very good. So, you thought of a different way. Instead of adding to six to one side, you could take away six from the other and it would balance.

7. Using a drawstring bag to represent the unknown

Following the initial work in small groups, the pan balance was subsequently used with a drawstring bag, a useful tool for providing a visual representation of the unknown and illustrating that the unknown must make both sides of the equals sign equal in value. The following dialogue explains how the teacher conveyed the role of the drawstring bag as representing something unknown. We used marbles to represent the quantities in the equation. In the problem situation referred to in the dialogue below, $8 = \square + 3$, we had already placed five marbles in the drawstring bag to ensure that the quantities would balance on the pan balance.

Teacher: Did you ever see a problem like this [*writing on the board*]: $8 = \square + 3$? Eight equals something unknown plus three.
Children: Yes [*nodding in agreement*].
Teacher: OK. I am going to show you here on my scales how to think about the "something unknown" [*placing marbles on the left side of the scale, one by-one*]. We put 1, 2, 3, 4, 5, 6, 7, 8 in here [*placing marbles on the right side of the scale, one by one*]. And I am going to put 1, 2, 3 on this side. Is the scale balanced?
Children: No.
Teacher: Now I am going to put my unknown on this side [*placing the drawstring bag on the right side of the scale*].
Teacher: What happened when I added my unknown?
Children: It balances.
Teacher: Yes. So, we don't know what is in the bag. How will we work it out?

8. Representing and solving word problems

We extended the pan balance work by presenting word problems. We wanted to provide opportunities for students to translate between problem scenarios and their algebraic representation. We encouraged the children to represent the problems as equations and represent unknown quantities with frames: $\square$. We gave them pan balances to explore solutions to the problem. Several groups solved the problems mentally and then used the pan balances to check their solutions, whereas other groups used pan balances from the outset. The children demonstrated flexibility in writing equations and were able to come up with several ways to algebraically represent any one scenario. See the **sidebar** above for examples of problem scenarios that might work well in your class. (The problems increase in complexity.)

9. True or false statements

As an assessment strategy at the completion of the sequence of instruction, we used true/false

Algebraic problem scenarios

1. Mr. Mahon's fourth-grade class had 42 students in it. A few more children joined the class in January. Then 50 students were in the class. How many joined the class in January?
   $42 + \square = 50$ or $\square + 42 = 50$

2. There were 30 children on a bouncing castle. Some of these children went inside to have a drink. Then 16 children were left on the bouncing castle. How many children went inside to have a drink?
   $30 - \square = 16$ or $16 + \square = 30$

3. Brad and his friends took part in a local charity cycling race between Magnolia Elementary School and Sunflower Elementary School. The schools are 10 miles apart. The group took 2 hours to complete the circuit. What was the cyclists' average speed?
   $10 = \square \times 2$ or $\square = 10 \div 2$

number sentences. Each child had a card with the word *true* printed on one side and *false* on the other side. We presented a variety of equations, and challenged the children to ascertain whether the statements were true or false. Here are some of the problem statements we used:

9 + 14 + 4 = 27
40 – 17 = 21
11 + 16 = 17 + 9
12 + 28 = 5 × 8

When the children had had enough time to make a decision regarding the accuracy of the statements, they used the true/false card to indicate their conclusion. In each case, students were asked to share their reasoning. Performance on the equations indicated whether the child had developed a relational understanding of equality (Molina and Ambrose 2006) and gave us an indication whether we had met the objective of the sequence of instruction (i.e., the development of a relational understanding of equality).

How can we demystify equation writing?

Developing relational understandings of equality in elementary school is not simple. Use a variety of teaching approaches to foster conceptual understanding of equality and to lay appropriate foundations for later work in equations and inequalities. The approaches presented in this article were sequential in nature, each one building on the children's previous notion of the concept of equality and extending it. For example, whereas the initial activities encouraged students to informally explain or show how both sides of a number sentence could be balanced (e.g., coins, a human number line), the children gradually gained increasing opportunities to create number sentences using frames to represent the unknown. The teaching approaches presented provide a variety of different ways (visual, kinesthetic, narrative, symbolic) for children to develop a relational understanding of equality, thus catering to the needs of all learners. In particular, the pan balance explorations highlighted that creating number sentences is an open-ended activity as well as the inverse relationship between addition and subtraction (see Teaching approach 6, Using pan balances). The various opportunities to explore the balancing process provided a springboard from which children were at ease in representing a single scenario in several different algebraic forms, thus demystifying the activity of writing equations.

Common Core Connections

1.OA.D.7
1.OA.D.8
3.OA.A.4

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